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Reactive power allocation through the modified Z-bus and Aumann-Shapley method

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ABSTRACT

This paper presents a new method based on the circuit theory and game theory for the allocation of reactive power. The allocation is calculated for each load, identifying and quantifying the responsibility of each reactive source. In the proposed method: the generators, line shunt, and bus shunt are modeled as current sources and loads are modeled as constant admittance, and obtained modified Z-bus matrix using circuit theory, which was coupled to the Aumann-Shapley method for calculating the unitary participation of each current source in the reactive power consumed by each load, considering each one as an independent player of the "reactive power allocation" game. The properties of the Aumann-Shapley method ensure equitable allocation and recovery of the total reactive power. Numerical results applied to the 5-bus and IEEE 30-bus systems are presented, discussed and compared with the other methods to demonstrate the applicability of the proposed method.

1. Introduction

In a traditional electricity market, the generation, transmission, and distribution of electric energy, as well as the ancillary services used for the reliable operation of the power system, were the responsibility of only one company. However, due to the appearance of competition in the electricity market, different regulatory frameworks emerged and with that, different types of markets where the generation, transmission, distribution and in some cases, commercialization is handled by different companies [1].

The transmission system services in a deregulated environment provide non-discriminatory open access by all agents and guarantee the means for the energy transference from the generations to the consumers, considering the technical restrictions and reliability criteria. To fulfill this purpose, the transmission system requires a series of additional services, which are called "ancillary services". The ancillary services are commonly used to ensure it is always possible to balance the supply and demand for energy in real-time [2]. The ancillary services may include several different operations which include reactive power support, spinning reserves, energy balance, frequency support, system restoration, etc [3].

In the case of reactive power support acting an ancillary service, it has two functions: a) to maintain the profile voltage within the technical limits and b) to maintain a reserve margin of reactive power to be used in cases of emergency [4].

In some countries is established that the independent system operator (ISO) is responsible for the coordination of voltage control and reactive power reserve. In order to maintain system reliability and security by providing for ancillary services such as reactive power support, the ISO identifies a set of reactive power requirements and looks for suitable providers, which are either generator and transmission companies. Usually, the ISO enters into bilateral contractual agreements with reactive power supplier for the procurement of this service [2], [3]. However in other countries, the reactive power support is the responsibility of the System Operator (SO) and can be supplied by generating companies and/or transmission companies [4,5].

Adequate reactive power support ensures the continuous supply of electricity, keeping minimal energy losses through the transmission lines. Therefore, the costs of these services must be adequately calculated and compensated by all the agents that make use of the system. To calculate the financial compensation for the reactive power support. The first step in this task is to identify what part of the reactive power support required is supplied by a specific generator, and what part is supplied by a specific reactive support device. However, to answer this

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question is very complicated due to the non-linear nature and non-separable nature of the power flow equations.

There are many methods in the scientific literature for the allocation of the cost of the reactive power support [6], therefore, a general description of some of them will be provided, highlighting its advantages and disadvantages.

The methods based on marginal cost use the concept of spot price to calculate the cost of reactive power support. The spot price is considered one of the most efficient ways to calculate the price of electricity because it optimizes social welfare. However, it has the following disadvantages: a) the spot price is very sensitive to the operating conditions of the system, causing fluctuations in the price of electricity in the short term, b) it depends on the method for the solution of the optimal power flow and may require more time and more computational resources for its convergence and finally, c) it does not allow recovering the total costs and hence additional charges called "complementary chargesg are required [7,8].

The methods based on power flow tracing generally use the proportional sharing principle and are considered very efficient when they are applied to the tracing of active power, because they allow determining from where, from whom and what part of the energy flow is consumed by a certain load. However, when applied for reactive power flow, they face two disadvantages: the first one, for technical reasons, reactive power cannot be transported over long distances [3], the second one, the reactive power does not always flow in the same direction as the active power, appearing bidirectional flows in the same branch, causing disruptions in the tracing of reactive power flow [9].

The methods based on circuit theory use the power flow solutions to calculate which part of the active and reactive power consumed by a load is supplied by each generator. These methods are considered efficient and reliable for applications in real systems since they are consistent with the network topology and the Kirchhoff's Laws. Among the most used methods are the Z-bus matrix [10], modified Y-bus matrix [11] and improved Y-bus matrix [12].

The main disadvantage of the Z-bus and Y-bus methods is the necessity of an additional process for allocating the participation in the reactive power consumed by a load. For example, in the method proposed by Chu et al [11], it uses the modified Y-bus and the proportional sharing principle. Chu considers the load bus voltages as a linear combination of the generator voltages, this causes two problems: a) cross-subsidies problems and b) the problem of allocation of participation to generators in loads without reactive power demand [13].

In the methods based on the cooperative game theory, the agents of the electric system form coalitions in order to minimize the transmission service costs. The main problem of this method is that the value assigned depends on the entry order into the coalition; therefore, the assigned value to an agent is different if it is entered first or last. This difficulty was overcome by the Shapley Value Method, in which the costs are assigned using the calculated cost average for every single possible combination of entry orders [14]. The main disadvantage of the Shapley Value is the use of computational resources. Nevertheless, by using the Aumann-Shapley Method the costs can be calculated through an analytic solution that is easy to implement and using less computational resources [15]. In general terms, the methods based on game theory, specifically the Aumann-Shapley Method, can recover the total costs and these are also considered fair and transparent.

The previously reported methods only consider reactive support devices and generators as reactive power sources, ignoring the reactive power injected by the line shunt and bus shunt. This leads to an error in the reactive power allocation process. For a fair allocation, in this paper, a new method was developed to solve the reactive power allocation problem considering the reactive power injected by the line shunt and bus shunt.

The proposed method is applied considering a known operation point. To determine the participation in the reactive power support of the generators, line shunts and bus shunts are modeled as current source, while loads are modeled as constant admittance. By obtaining the modified Z-bus matrix, which will be coupled in the Aumann-Shapley method for calculating the unitary participation of each generator, bus shunt and the line shunt in the reactive power consumed by each load.

The main contributions of the proposed method follow:

1. It considers circuit laws and at the same time has desirable characteristics in terms of economic coherence, because it is based on circuit laws in combination with Aumann-Shapley method.
2. It identifies and quantifies the individual contribution of each agent (generator, load, bus shunt and line shunt), even when connected to the same bus.
3. It fully allocates the reactive power (full cost recovery), thanks to the additive property of the Aumann-Shapley method
4. It is considered fair and transparent, given that it is based on circuit theory in combination with the cooperative game theory (Aumann Shapley Method).
5. It does not require much computational effort, since it is based on an analytical method (Aumann-Shapley Method), making it a strong candidate for real-time applications.

This paper is organized as follows. The mathematical model for the proposed method is presented in Section 2. The numerical solutions obtained by applying the proposed method with the 5-bus system and IEEE 30-bus system are reported and discussed in Section 3. Finally, conclusions, commentaries and final considerations are presented in Section 4.

2. Proposed method

2.1. Background

By applying the Kirchhoff's Laws in a system with n buses at a known operation point, the following expression is obtained.

$$I = YE \tag{1}$$

Where I is the nodal current injection vector, Y is the nodal admittance matrix and E is the nodal voltage vector.

For the purposes of enforcement of the proposed method, the line shunt and bus shunt are modeled as reactive power sources, and therefore they have a participation in the reactive power consumed by each load.

In Fig. 1, the simplified schematic is shown for a 2-bus system. The generator is represented by a current source (I_K^G), in the transmission line, the line shunt is represented by I_{km}^{Sh} and I_{mk}^{Sh}, positioned in the ends of the line; the bus shunt is represented by I_m^{SB}, connected to the load bus and the load is represented by a constant admittance Y_m^L.

On the other hand, if distributed generation sources (DGs) are considered. In the proposed method it can be modeled as current sources that can inject or consume reactive power. Therefore, for a known system operating point, a DG can be modeled as a conventional generator.

The equivalent admittance of the loads are incorporated into the elements in the diagonal of the Y-matrix, obtaining the modified Y-bus matrix. Thus, the Eq. (1) is modified.

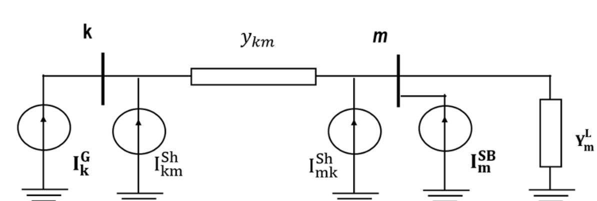

Fig. 1. 2-Bus System Model with the generators, line shunt and bus shunt.

$$I' = Y'E \tag{2}$$

Where I' is the current injection vector, Y' is the modified admittance matrix (including the load admittance) and E is the nodal voltage vector. Therefore, the element (i,j) of the modified admittance matrix is obtained from the element (i,j) of the nodal admittance matrix.

$$Y_{ij}' = \begin{cases} Y_{ij} + Y_j^L, & \text{for } i = j \\ Y_{ij}, & \text{for } i \neq j \end{cases} \tag{3}$$

The equivalent admittance of the load L connected to the bus j, can be calculated as:

$$Y_j^L = \frac{1}{E_j}\left(\frac{S_j^L}{E_j}\right)^* \tag{4}$$

Where E_j and S_j^L are the voltage and apparent power consumed by the load L connected to the bus j. Both values are obtained by the power flow.

By inverting the modified admittance matrix shown in the Eq. (2), the modified impedance matrix Z' is obtained.

$$\begin{bmatrix} E_1 \\ \vdots \\ E_k \\ \vdots \\ E_n \end{bmatrix} = \begin{bmatrix} Z_{11}' & \cdots & Z_{1k}' & \cdots & Z_{1n}' \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{k1}' & \cdots & Z_{kk}' & \cdots & Z_{kn}' \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{n1}' & \cdots & Z_{nk}' & \cdots & Z_{nn}' \end{bmatrix} \begin{bmatrix} I_1' \\ \vdots \\ I_k' \\ \vdots \\ I_n' \end{bmatrix}$$

Considering that the bus k is connected to the load L. From the previous matrix, the load voltage can be calculated in a linear combination of currents injected into the system.

$$E_k = Z_{k1}'I_1' + \cdots + Z_{kk}'I_k' + \cdots + Z_{kn}'I_n' = \sum_{j=1}^{n} Z_{kj}'I_j' \tag{5}$$

Where Z_{kj}' is the element (k,j) of the modified impedance matrix and I_j' is the element j of the current injection vector.

In general terms, considering the $k-th$ bus, the total current injected into that bus will be calculated by the following equation:

$$I_k' = I_k^G + I_k^{SL} + I_k^{SB} \tag{6}$$

Where I_k^G is the current injected by the generator into the bus k; I_k^{SL} and I_k^{SB} are the current injected by the line shunt and bus shunt into the bus k, respectively.

Supposing that the $k-th$ bus is connected to b buses, the total current injected by the line shunts into the bus k will be calculated as following:

$$I_k^{SL} = \sum_{j=1}^{b} I_{kj}^{Sh} \tag{7}$$

Where I_{kj}^{Sh} is the current injected by the line shunt between the bus k and the bus j.

The apparent power consumed by the load L connected to the bus k, can be calculated as follows:

$$S_k^L = \left(Y_k^L\right)^*|E_k|^2 \tag{8}$$

Where Y_k^L is the load admittance connected to the bus k, and E_k, is the load voltage.

Considering $I_i' = I_i'^r + jI_i'^i$ and $Z_{ki}' = R_{ki}' + jX_{ki}'$, the Eq. (5) becomes the following expression.

$$E_k = \sum_{i=1}^{n}\left(R_{ki}'I_i'^r - X_{ki}'I_i'^i\right) + i\sum_{i=1}^{n}\left(R_{ki}'I_i'^i + X_{ki}'I_i'^r\right) \tag{9}$$

Replacing the Eq. (9) in (8) it is obtained as follows.

$$S_k^L = Y_k^L\left[\left(\sum_{i=1}^{n}\left(R_{ki}'I_i'^r - X_{ki}'I_i'^i\right)\right)^2 + \left(\sum_{i=1}^{n}\left(R_{ki}'I_i'^i + X_{ki}'I_i'^r\right)\right)^2\right] \tag{10}$$

Considering $S_k^L = P_k^L + jQ_k^L$ and $Y_k^L = g_k^L + jb_k^L$, it can be obtained a mathematical expression for the reactive power consumed by each load.

$$Q_k^L = -b_k^L\left[\left(\sum_{i=1}^{n}\left(R_{ki}'I_i'^r - X_{ki}'I_i'^i\right)\right)^2 + \left(\sum_{i=1}^{n}\left(R_{ki}'I_i'^i + X_{ki}'I_i'^r\right)\right)^2\right] \tag{11}$$

2.2. Reactive power injected by line shunt and bus shunt

The total reactive power injected by the line shunt into the bus k can be calculated as follows:

$$Q_k^{SL} = \sum_{m=1}^{b}\left(I_{km}^{sh}\right)\cdot E_k \tag{12}$$

Where b is the number of buses connected to bus k, E_k, voltage in bus k, I_{km}^{Sh}, current injected by the line shunt, specifically into the bus k.

The injection of reactive power by the bus shunt in bus k can be calculated using the following equation:

$$Q_k^{SB} = \left(I_k^{SB}\right)\cdot E_k \tag{13}$$

Where I_k^{SB} is the current injected by the bus shunt located at bus k, and E_k, is voltage in bus k.

The total reactive power injected by the line shunt and the buse shunt into bus k is estimated as follows:

$$Q_k^S = Q_k^{SB} + Q_k^{SL} \tag{14}$$

Before starting the reactive allocation process on each bus system, a comparison is made between the reactive power injected by the line shunt and the bus shunt (Q_k^S) and the reactive power consumed by the load (Q_k^L). If Q_k^S is higher than Q_k^L, the excess of reactive power in such bus is allocated among the remaining system loads. If, on the other hand, Q_k^S is lower than Q_k^L, the lack of reactive power is supplied by the other reactive power sources.

2.3. Reactive power allocation through the Aumann-Shapley

The Aumann-Shapley Method allows, fairly and transparently, to solve different allocation problems. For this reason, in this article is used the Aumann-Shapley method in combination with circuit theory to solve the problem of reactive power allocation. For more information about the properties of the Aumann-Shapley Method you can see [15–18].

The proposed method allows calculating how each current source participates in the reactive power consumed by each load. Each current source as an independent player in the reactive power allocation game.

From the Eq. (11) we can see that the power consumed by the load L connected to the bus k is a linear combination of all the currents injected into the system (generators, line shunt and bus shunt). Furthermore, the Aumann-Shapley method can be utilized to determine the responsibility of each current source in the reactive power consumed by each load.

The Aumann-Shapley Method consists of dividing each current source into infinitesimal parts of the same size; considering each one of them as single players. This provides an analytic solution to the reactive power allocation problem [19].

For a system with n buses, there are n players$[I_1', \ldots, I_k', \ldots, I_n']$ or $2n$ players, if the real and imaginary current sources components $[I_1'^r, I_1'^i, \ldots, I_k'^r, I_k'^i, \ldots, I_k'^r, I_k'^i]$ are analyzed. For example, the total current injected into the bus x is considered as $I_x' = I_x'^r + jI_x'^i$.The unitary participation of the real component ($I_x'^r$) in the reactive power consumed by the load L connected with the bus k can be calculated through the next equation.

$$PU_{I_x^{'r}\to L_k} = \int_0^1 \frac{\partial Q_k^L(I^{'}t)}{\partial I_x^{'r}} dt \tag{15}$$

Substituting the value of Q_k^L of the Eq. (11) into the Eq. (15) after derive and integrate, the following equation is obtained.

$$PU_{I_x^{'r}\to L_k} = -b_k^L\left[\sum_{i=1}^{n}\left(R_{ki}^{'}I_i^{'r} - X_{ki}^{'}I_i^{'i}\right)R_{kx}^{'} + \sum_{i=1}^{n}\left(R_{ki}^{'}I_i^{'i} + X_{ki}^{'}I_i^{'r}\right)X_{kx}^{'}\right] \tag{16}$$

The unitary participation of the imaginary component ($I_x^{'i}$) in the reactive power consumed by the load L connected to the bus k can be calculated through the next equation.

$$PU_{I_x^{'i}\to L_k} = \int_0^1 \frac{\partial Q_k^L(I^{'}t)}{\partial I_x^{'i}} dt \tag{17}$$

Substituting the value of Q_k^L of the Eq. (11) into the Eq. (17) after derive and integrate, the following equation is obtained.

$$PU_{I_x^{'i}\to L_k} = -b_k^L\left[\sum_{i=1}^{n}\left(R_{ki}^{'}I_i^{'r} - X_{ki}^{'}I_i^{'i}\right)\left(-X_{kx}^{'}\right) + \sum_{i=1}^{n}\left(R_{ki}^{'}I_i^{'i} + X_{ki}^{'}I_i^{'r}\right)R_{kx}^{'}\right] \tag{18}$$

The total participation of the current injected into the bus x in the reactive power consumed by the load L connected to the bus k is obtained through the next equation:

$$PT_{I_x^{'}\to L_k} = I_x^{'r}\cdot PU_{I_x^{'r}\to L_k} + I_x^{'i}\cdot PU_{I_x^{'i}\to L_k} \tag{19}$$

According to the Eq. (6), the total current injected into the bus x is composed of the current injected by the generator ($I_x^G = I_x^{G_r} + jI_x^{G_i}$), the current injected by the line shunt ($I_x^{SL} = I_x^{SL_r} + jI_x^{SL_i}$) and the current injected by the bus shunt ($I_x^{SB} = I_x^{SB_r} + jI_x^{SB_i}$). Therefore:

$$I_x^{'} = I_x^{'r} + jI_x^{'i} = \left(I_x^{G_r} + I_x^{SL_r} + I_x^{SB_r}\right) + j\left(I_x^{G_i} + I_x^{SL_i} + I_x^{SB_i}\right) \tag{20}$$

Replacing the components from the Eq. (20) into (19).

$$PT_{I_x^{'}\to L_k} = \left(I_x^{G_r} + I_x^{SL_r} + I_x^{SB_r}\right)\cdot PU_{I_x^{'r}\to L_k} + \left(I_x^{G_i} + I_x^{SL_i} + I_x^{SB_i}\right)\cdot PU_{I_x^{'i}\to L_k} \tag{21}$$

By arranging the terms of the Eq. (21), it can be obtained that the total participation of the current injected into the bus x in the reactive power consumed by the load L connected with the bus k is the sum of the individual participation of each current source, therefore:

$$PT_{I_x^{'}\to L_k} = PTG_{I_x^{'}\to L_k} + PTSL_{I_x^{'}\to L_k} + PTSB_{I_x^{'}\to L_k} \tag{22}$$

Where:

$PTG_{I_x^{'}\to L_k}$: Total participation of the generator connected to the bus x in the reactive power consumed by the load connected to the bus k.
$PTSL_{I_x^{'}\to L_k}$: Total participation of the line shunt connected to the bus x in the reactive power consumed by the load connected to the bus k.
$PTSB_{I_x^{'}\to L_k}$: Total participation of the bus shunt connected to the bus x in the reactive power consumed by the load connected to the k.

The total participation of each current source is calculated according to:

$$PTG_{I_x^G\to L_k} = I_x^{G_r}\cdot PU_{I_x^{'r}\to L_k} + I_x^{G_i}\cdot PU_{I_x^{'i}\to L_k} \tag{23}$$

$$PTSL_{I_x^{SL}\to L_k} = I_x^{SL_r}\cdot PU_{I_x^{'r}\to L_k} + I_x^{SL_i}\cdot PU_{I_x^{'i}\to L_k} \tag{24}$$

$$PTSB_{I_x^{SL}\to L_k} = I_x^{SB_r}\cdot PU_{I_x^{'r}\to L_k} + I_x^{SB_i}\cdot PU_{I_x^{'i}\to L_k} \tag{25}$$

Eqs. (23), (24) and (25), show that the total participation of the generator, the line shunt and the bus shunt depend on the real and imaginary components of the current injected by each source. Furthermore, from the Eqs. (24) and (25), it can be shown, that although the line shunt and bus shunt are considered reactive power sources, they have a real component in the current injected into the system.

3. Case studies

The proposed method for the reactive power allocation was implemented and tested in two base cases: the 5-bus test system and the IEEE 30-bus system. The results obtained by the proposed method (PM) were compared with other methods such as Allocation Method Via Voltage Sources (AMV) [13] and Modified Y-bus Matrix Method (MYM)[11].

3.1. 5-Bus system

The 5-bus system presented in [13], was used to test the proposed method (see Fig. 2). The system contains 3 generators, the first one is located in bus 1 (G_1), the second one is located in bus 3 (G_3) and the third one is located in bus 4 (G_4); 5 loads located in the buses 1 to 5 and 6 transmission lines.

From the power flow results, shown in Table 1, it can be observed that: the active power delivered by the generators G_1 and G_4 is much bigger than their reactive power. This will have a strong influence in the reactive power allocation process in the proposed method due to the coupling effect that exists between active and reactive power [18].

Table 2 shows the numerical results of the reactive power allocated to each generator with the 5-bus test system considering the AMV and MYM methods and the proposed method (PM). Based on the results shown in Table 2, the following statements can be concluded.

- The values of reactive power allocated to the generators have different values for each of the three methods being compared in this paper.
- The reactive power consumed by loads 1, 3 and 4 are directly supplied by generators G_1, G_3 and G_4, respectively.
- The AMV and MYM methods do not consider some generators. For example, based on methods AMV and MYM, the reactive power consumed by the load 2 is supplied by generators G_1 and G_3. However, through the proposed method, in addition to the reactive power allocated to generators G_1 and G_3, the reactive power consumed by the load 2 has a participation of 5.7703MVAr allocated to the generator G_4.
- Based on the proposed method, the reactive power consumed by the load 2 is supplied by generator G_1 (10.8132MVAr), generator G_3 (3.4165MVAr) and generator G_4 (5.7703MVAr). Although the electric distance of generator G_4 is farther from load 2 than generator G_3 [13], it has more participation in the reactive power consumed by this load. This is mainly because the generators are modeled as complex current injections. As a result, the real current component,

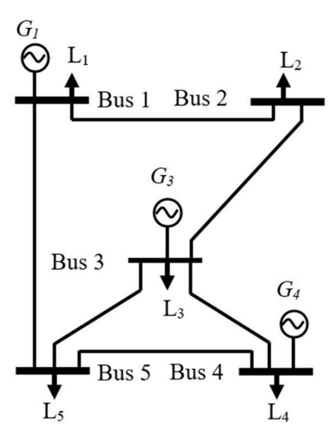

Fig. 2. 5-bus test system.

Table 1
Power Flow Results for 5-bus test system.

Bus	Voltage		Generator		Load	
	V(pu)	θ (grad)	P(MW)	Q(MVAr)	P(MW)	Q(MVAr)
1	1.05	0	239.96	45.27	45	15
2	0.989	-10.8	0	0	162.5	20
3	1.033	-7.8	62.3	66.68	80	20
4	1.05	0.47	136.68	22.16	50	20
5	1.015	-4.82	0	0	90	25

as well as the imaginary current component injected by the generator, have participation in the total reactive power allocated to each generator. As shown in Table 3, approximately 96% (5.5496MVAr) of the reactive power supplied by the generator G_4 to cover the reactive power consumed by the load 2 is supplied by the real current component, and approximately 4% (0.2207MVAr) is supplied by the imaginary component. This is due to the coupling effect between active and reactive power [18]. This means that the active power consumed by a load is supplied simultaneously by the active and reactive power of each generator. Similarly, the reactive power consumed by a load is supplied by the reactive and active power of each generator [20].

To demonstrate the procedure of the proposed method, two bus shunts were added to the 5-bus test system shown in figure 2: one was added to the bus 2 ($b_2^{sh} = 0.15pu$) and the other one to the bus 5 ($b_5^{sh} = 0.10pu$). Both, the line shunts and the bus shunts will have a participation in the reactive power consumed by each load. To calculate the participation of generators, line shunts and bus shunts in the reactive power consumed by each load with the proposed method, they are modeled as current sources.

The results of the power flow solution considering the bus shunts added to the system are shown in Table 4. When comparing the values shown in Table 4 against the values shown in Table 1, two differences are observed: the first is that voltage of buses 2 and 5 increases due to an injection of reactive power by the bus shunt connected to those buses. The second difference is the decrease of reactive power injected by each generator.

Table 5 shows the results of allocating reactive power considering the generators, the bus shunts and the line shunts as current sources. Based on the results shown in Table 5, the following statements can be concluded:

- Based on the Table 2, 100% of the reactive power consumed by load 1 is supplied by generator G_1. However, based on the Table 5 when considering the effect of the line shunt as a source of reactive power, 14.7% of the reactive power consumed by load 1 is supplied by the line shunt and 85.3% is supplied by generator G_1.
- The reactive power consumed by load 5 is supplied as follows: 41.73%, by the bus shunt connected to bus 5; 7.51%, by the line shunts; 27.74%, by generator G_1; 7.44%, by generator G_3 and 15.58%, by generator G_4.

3.2. IEEE 30-Bus system

The IEEE 30-bus system was utilized to prove the proposed method. It is composed of 6 generators connected to bus 1 (G_1), bus 2 (G_2), bus 5 (G_5), bus 8 (G_8), bus 11 (G_{11}) and bus 13 (G_{13}), respectively. Twenty loads and forty-one transmission lines. Detailed information about this system can be found in [13].

To better understand the effect of reactive power allocation to the generators of the IEEE 30-bus system when considering bus shunts and line shunt as reactive power sources, two cases are considered: case I, when line shunts and bus shunts are considered part of the nodal admittance matrix; case II, when bus shunts and line shunt are modeled as current sources.

3.2.1. Case i

The results of allocating reactive power with the AMV and MYM methods as well as the proposed method (PM) in case I, are shown in Table 6. When comparing the results obtained by each three methods presented in this paper, reported in Table 6, the following differences can be noted:

- Bus 10 is a load that only consumes active power; therefore, the AMV and MYM methods, as well as the proposed method, should not be considered for the allocation of reactive power. However, method MYM assigns participation to all generators available in the system. Although the sum of the participation of each generator in bus 10 is zero, as shown in Table 6, it results in cross-subsidies.
- The MYM method allows the possibility to allocate negative values to the reactive power injected by the generators. For example, with the

Table 3
Total participation of the real and imaginary current component injected by each generator.

bus	Generador	$I_x^{G_r} \cdot PU_{I_x^{'r} \to L_k}$	$I_x^{G_i} \cdot PU_{I_x^{'i} \to L_k}$	$PTG_{I_x^G \to L_k}$	
			MVAr	MVAr	MVAr
	2	G_1	10.1128	0.7004	10.8132
L		G_2	2.2612	1.1553	3.4165
O		G_3	5.5496	0.2207	5.7703
A	5	G_1	12.7719	0.8134	13.5853
D		G_2	2.799	1.0403	3.8393
		G_3	7.2006	0.3747	7.5753

Table 4
Power flow calculated by adding the bus shunt .

Bus	Voltage		Generator		Load	
	V(pu)	θ (grad)	P(MW)	Q(MVAr)	P(MW)	Q(MVAr)
1	1.05	0	239.47	33.36	45	15
2	1.001	-10.83	0	0	162.5	20
3	1.033	-7.72	62.3	54.43	80	20
4	1.05	0.51	136.68	19.58	50	20
5	1.021	-4.86	0	0	90	25

Table 2
Results of the allocation of reactive power.

Bus	Voltage	Supplied by G_1 (MVAr)			Supplied by G_3 (MVAr)			Supplied byG_4 (MVAr)			Load
N	V(pu)	AMV	MYM	PM	AMV	MYM	PM	AMV	MYM	PM	
1	1.05	15	15	15	0	0	0	0	0	0	15
2	0.989	8.6404	14.2249	10.8132	11.3596	5.7751	3.4165	0	0	5.7703	20
3	1.033	0	0	0	20	20	20	0	0	0	20
4	1.05	0	0	0	0	0	0	20	20	20	20
5	1.015	11.9829	13.4419	13.5853	7.0341	4.5883	3.8394	5.983	6.9698	7.5753	25

Table 5
Allocation of reactive power considering line shunts and bus shunts.

Bus	Supplied by Generator (MVAr)			Supplied by line Shunt (MVAr)							Supplied by Bus Shunt	Load
	G_1	G_3	G_4	L_{1-2}	L_{1-5}	L_{2-3}	L_{3-4}	L_{3-5}	L_{4-5}	Total		
1	12.795	0	0	1.6537	0.5513	0	0	0	0	2.205	0	15
2	1.3518	0.401	0.729	1.5016	0	1.001	0	0	0	2.5026	15.0156	20
3	0	17.4923	0	0	0	1.0671	0.6403	0.8003	0	2.5077	0	20
4	0	0	18.7321	0	0	0	0.6615	0	0.6064	1.2679	0	20
5	6.9353	1.8592	3.8948	0	0.5216	0	0	0.7825	0.5738	1.8779	10.4328	25

MYM method the reactive power consumed by load 4 is supplied by G_1, G_2, G_5, G_8, G_{11} and G_{13}. Generators G_1, G_2, G_5 and G_8, are assigned positive values of reactive power, while generators G_{11} and G_{13}, are assigned negative values of reactive power. This may benefit to some generators, and work against some others, thus resulting in cross-subsidies.

- According to AMV and MYM methods, the total power injected by generator G_8 is 47.3379MVAr and 47.0114MVAr, respectively. However, according to the power flow, generator G_8 only injects a reactive power of 18.07 MVAr to the system. The allocation of reactive power is higher than the reactive power injected to the system by generator G_8, which can only be explained by the coupling effect between the active and reactive power. In other words, the active power from generator G_8 has participation in the reactive power consumed by the loads.
- When comparing the values of reactive power allocated to the generators to cover the demand on load bus 7, it can be observed that the AMV method allocates more reactive power to generator G_5, because the electric distance of this generator is closer to that load [13]. However, more reactive power is allocated to generator G_1, with the proposed method due to the coupling effect between the active and reactive power [21]. It means that the increased value of reactive power allocated to generator G_1, results mainly from the active power injected into the system by this generator (60% of the total demanded by the IEEE 30-bus system).
- When comparing the total reactive power injected by each generator as shown in Table 6, it is observed with the PM, the generator G_1, injects more reactive power (27.8432MVAr) than AMV method (3.2680MVAr) and MYM method (4.2765MVAr). This is mainly due to the coupling effect between the active and reactive power.

The three methods presented in this article are based on the power flow solution. However, they allocate different values of reactive power to each generator. This is mainly due to the criteria chosen to develop the model. That is why there are no models of reactive power allocation universally accepted currently.

3.2.2. Case II

In case II, the bus shunts and the line shunts are modeled as reactive power sources. The results of the reactive power allocation in case II using the proposed method (PM) are shown in Table 7. No calculations were made using the AMV method or the MYM method because they do not include them in their mathematical approach. When comparing tables 7 and 6, the following can be observed:

- 60% of the reactive power demanded by load 2 is supplied by generator G_2 and the remaining 40% is supplied by the line shunts.
- The demands from loads 3 and 4 are directly covered by the line shunts; therefore, no reactive power is allocated to the generators
- The values of reactive power allocated to the generators decrease when considering the line shunts and bus shunts as sources of reactive power. For example, if we consider the demand from load 7, it can be observed that the value assigned to generator G_1 decreases from 3.6223MVAr to 2.5979MVAr.
- The reactive power consumed by load 30 is supplied by the bus shunt connected to that bus.
- As previously mentioned, for each system bus a comparison was made between the reactive power injected by the line shunt and the bus shunt, and the demanded reactive power. If the reactive power injected into the bus was higher than the demand, the bus was considered as a reactive power source; thus, it would be considered a new player in the reactive power allocation game (this is the case of the line shunts in bus 1 (B_1), bus 3 (B_3), bus 4 (B_4), bus 6 (B_6), bus 28 (B_{28}) and the bus shunt in bus 30 (B_{30})). For example, the reactive power consumed by load 7 is 7.5MVAr. The reactive power injected by all line shunts into the bus 7 is 1.8761MVAr, reducing the reactive power consumed to 5.6239MVAr. As a result, it will have the reduction of the participation of reactive power sources on this load bus.
- The total power injected by the line shunts equals to 15.9%, while the total power injected by the bus shunts are 1.6% of the total reactive power consumed by the system (130.8 MVAr).

According to the results shown in Table 7, the line shunts inject into the system a significant part of the reactive power, which must be considered in the allocation process by the reactive power support. If this is not taken into account, we fall into cross-subsidy problems, because some agents would be benefited from greater participation in the allocation of reactive power.

4. Conclusion

In this paper, a new method is proposed in order to determine the participation of reactive power sources (generators, bus shunts and line shunts) on reactive power consumed by loads, considering the coupling existing between active and reactive power.

The proposed method is based on circuit theory in combination with game theory, through the analytical application of Aumann-Shapley method. The properties of circuit theory and Aumann-Shapley axioms are combined to provide with appropriate and more precise allocation of reactive power.

The results obtained validated the influence of active and reactive components on the reactive power allocation process between agents, and the need for representing generators, bus shunts and line shunts as independent agents, including when they are at the same bus. Therefore, it can be concluded that reactive power participation in loads cannot be separated from active and reactive components, and an integrated approach is required.

The Proposed Method demonstrated that line shunts injected a significant part of reactive power that must be considered in the process of reactive power allocation, thus avoiding cross-subsidy problems.

CRediT authorship contribution statement

Carlos Castillo C: Conceptualization, Methodology, Software, Formal analysis, Writing - original draft. **Yuri Molina:** Conceptualization, Methodology, Software, Validation, Supervision, Formal analysis. **Jaime Luyo:** Conceptualization, Methodology, Supervision. **Raoni Pegado:** Software, Validation.

Table 6
Results of the allocation of reactive power for the IEEE 30-bus system (case I).

Bus	Supplied by G_1 (MVAr)			Supplied by G_2 (MVAr)			Supplied by G_5 (MVAr)			Supplied by G_8 (MVAr)			Supplied by G_{11} (MVAr)			Supplied by G_{13} (MVAr)			Load
	AMV	MYM	PM	AMV	MYM	PM	AMV	MYM	PM	AMV	MYM	PM	AMV	MYM	PM	AMV	MYM	PM	
2	0	0	0	22.7	22.7	22.7	0	0	0	0	0	0	0	0	0	0	0	0	22.7
3	0.3787	0.4208	0.5951	0.2775	0.3346	0.2502	0.0635	0.0654	0.1093	0.3581	0.3513	0.1088	0.0395	0.0125	0.0645	0.0828	0.0154	0.0721	1.2
4	0.2653	0.3988	0.7813	0.4507	0.6808	0.333	0.1032	0.1129	0.1474	0.5819	0.5683	0.1476	0.0642	-0.0385	0.0898	0.1347	-0.1223	0.101	1.6
5	0	0	0	0	0	0	19	19	19	0	0	0	0	0	0	0	0	0	19
7	0.2773	0.376	3.6223	0.8559	1.1449	1.5505	3.4922	3.6163	0.7786	2.443	2.3881	0.6929	0.226	-0.0287	0.4171	0.2056	0.0034	0.4386	7.5
8	0	0	0	0	0	0	0	0	0	30	30	30	0	0	0	0	0	0	30
10	**0**	**0.0416**	**0**	**0**	**0.0828**	**0**	**0**	**-0.0001**	**0**	**0**	**-0.0217**	**0**	**0**	**-0.222**	**0**	**0**	**0.1195**	**0**	**0**
12	0.4026	0.4839	3.4773	0.7594	0.9377	1.4633	0.2222	0.2577	0.651	1.3	1.4795	0.659	0.5811	0.6474	0.4719	4.2347	3.6938	0.7774	7.5
14	0.0842	0.1281	0.7429	0.1652	0.2643	0.3128	0.0519	0.0714	0.1394	0.3078	0.4057	0.1412	0.1568	0.1829	0.1031	0.8341	0.5476	0.1606	1.6
15	0.1289	0.1888	1.163	0.2632	0.3882	0.4898	0.088	0.1048	0.2186	0.5283	0.5952	0.2218	0.2947	0.2498	0.1649	1.1969	0.9732	0.2418	2.5
16	0.0895	0.1154	0.8382	0.1926	0.2451	0.3532	0.0692	0.0744	0.1579	0.4067	0.4265	0.1604	0.2858	0.2304	0.1245	0.7562	0.7082	0.1658	1.8
17	0.2663	0.3319	2.7118	0.653	0.7828	1.1445	0.2734	0.2762	0.5142	1.6104	1.5962	0.5239	1.3855	1.0928	0.4363	1.6113	1.7201	0.4693	5.8
18	0.0441	0.0674	0.4197	0.0978	0.1454	0.1769	0.0366	0.0407	0.0792	0.2182	0.23	0.0805	0.1552	0.0993	0.0631	0.3481	0.3172	0.0805	0.9
19	0.1614	0.2303	1.5877	0.3766	0.5159	0.6698	0.1496	0.1575	0.3004	0.8879	0.8989	0.3057	0.698	0.4658	0.247	1.1266	1.1317	0.2895	3.4
20	0.0327	0.0486	0.3271	0.0783	0.1103	0.138	0.032	0.0333	0.062	0.1897	0.1892	0.0631	0.1558	0.0938	0.0518	0.2115	0.2248	0.058	0.7
21	0.5009	0.6261	5.2482	1.2929	1.5433	2.2168	0.5686	0.5693	0.9978	3.4158	3.3512	1.0183	2.9113	2.2796	0.8597	2.5106	2.8304	0.8592	11.2
23	0.0794	0.1022	0.7474	0.1765	0.2239	0.3152	0.0663	0.0705	0.1411	0.4157	0.4219	0.1437	0.2399	0.204	0.1092	0.6222	0.5775	0.1434	1.6
24	0.3146	0.3741	3.1477	0.7853	0.9097	1.3298	0.3347	0.3385	0.5979	2.1867	2.1365	0.6113	1.3001	1.1388	0.4784	1.7785	1.8025	0.535	6.7
26	0.1089	0.131	1.0932	0.2966	0.3426	0.4629	0.1366	0.1337	0.2089	1.0277	0.9464	0.2153	0.316	0.2903	0.1527	0.4142	0.456	0.167	2.3
29	0.0428	0.0573	0.4308	0.1226	0.153	0.1827	0.0587	0.0553	0.0826	0.4692	0.3932	0.0856	0.0921	0.0883	0.057	0.1146	0.1529	0.0613	0.9
30	0.0904	0.1542	0.9094	0.2588	0.3933	0.3856	0.1239	0.1088	0.1744	0.9906	0.6549	0.1807	0.1943	0.1779	0.1204	0.242	0.4108	0.1295	1.9
Total	3.268	4.2765	27.8432	29.8027	31.8987	34.4752	24.8705	25.0864	24.3607	47.3379	47.0114	35.3599	9.0963	6.9644	4.0111	16.4245	15.5626	4.7499	130.8

Table 7
Allocation of reactive power with the Proposed Method (PM) for the IEEE 30-bus system (case II).

Bus	Supplied by generator (MVAr)								Supplied by line shunts (MVAr)								Supplied by bus shunt (MVAr)	Load		
	G_1	G_2	G_5	G_8	G_{11}	G_{13}	Total	B_1	B_3	B_4	B_6	B_{28}	Q_k^{SL}	Total		B_{30}			Q_k^{SB}	Total
2	0	13.4724	0	0	0	0	13.4724		0	0	0	0	0	9.2276	9.2276		0	0	0	22.7
3	0	0	0	0	0	0	0		0	0	0	0	0	1.2	1.2		0	0	0	1.2
4	0	0	0	0	0	0	0		0	0	0	0	0	1.6	1.6		0	0	0	1.6
5	0	0	15.8377	0	0	0	15.8377		0	0	0	0	0	3.1623	3.1623		0	0	0	19
7	2.5979	1.1492	0.5845	0.498	0.3213	0.3395	5.4905		0.0487	0.0111	0.0094	0.034	0.0211	1.8761	2.0004		0.0091	0	0.0091	7.5
8	0	0	0	27.3579	0	0	27.3579		0	0	0	0	0	2.6421	2.6421		0	0	0	30
10	0	0	0	0	0	0	0		0	0	0	0	0	0	0		0	0	0	0
12	3.3272	1.4468	0.6532	0.6321	0.4815	0.7831	7.3239		0.0614	0.0151	0.013	0.0423	0.0269	0	0.1587		0.0174	0	0.0174	7.5
14	0.7106	0.3092	0.1398	0.1354	0.105	0.1619	1.5619		0.0131	0.0032	0.0028	0.0091	0.0058	0	0.034		0.004	0	0.004	1.6
15	1.112	0.4841	0.2192	0.2126	0.1679	0.244	2.4399		0.0205	0.005	0.0043	0.0143	0.0092	0	0.0534		0.0067	0	0.0067	2.5
16	0.8016	0.3491	0.1584	0.1538	0.1266	0.1676	1.757		0.0148	0.0036	0.0031	0.0104	0.0066	0	0.0385		0.0045	0	0.0045	1.8
17	2.5921	1.1308	0.5155	0.5017	0.4423	0.4764	5.6588		0.0478	0.0116	0.01	0.0343	0.022	0	0.1257		0.0155	0	0.0155	5.8
18	0.4012	0.1748	0.0794	0.0772	0.0641	0.0815	0.8782		0.0074	0.0018	0.0016	0.0052	0.0034	0	0.0194		0.0024	0	0.0024	0.9
19	1.5177	0.6617	0.3011	0.2929	0.2506	0.2933	3.3173		0.028	0.0068	0.0059	0.02	0.0128	0	0.0734		0.0093	0	0.0093	3.4
20	0.3127	0.1364	0.0621	0.0604	0.0525	0.0588	0.6829		0.0058	0.0014	0.0012	0.0041	0.0027	0	0.0152		0.0019	0	0.0019	0.7
21	5.0135	2.1892	0.9998	0.9745	0.8705	0.8738	10.9212		0.0926	0.0223	0.0192	0.0671	0.0434	0	0.2446		0.0342	0	0.0342	11.2
23	0.7138	0.3112	0.1414	0.1375	0.111	0.1449	1.5599		0.0132	0.0032	0.0028	0.0093	0.0061	0	0.0346		0.0055	0	0.0055	1.6
24	3.0012	1.3114	0.5984	0.5841	0.4851	0.5427	6.5228		0.0556	0.0134	0.0115	0.0401	0.0269	0	0.1476		0.0296	0	0.0296	6.7
26	1.0368	0.4547	0.2083	0.2048	0.1549	0.1697	2.2292		0.0193	0.0046	0.004	0.0142	0.0103	0	0.0525		0.0183	0	0.0183	2.3
29	0.4044	0.1782	0.0819	0.0808	0.0578	0.0623	0.8655		0.0076	0.0018	0.0016	0.0057	0.0043	0	0.021		0.0135	0	0.0135	0.9
30	0	0	0	0	0	0	0		0	0	0	0	0	0	0		0	1.9	1.9	1.9
Total	23.543	23.759	20.581	31.904	3.691	4.399	107.877		0.4359	0.1051	0.0903	0.3101	0.2016	19.7081	20.8511		0.1719	1.9	2.0719	130.8

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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